MULTIMEDIA UNIVERSITY

STUDENT ID NO

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**Table NO: (  )**

# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 1, 2022/2023 (TERM ID 2215)**

## TMA1201 – DISCRETE STRUCTURES AND PROBABILITY

(All sections / Groups)

16 FEBRUAY 2023
9.00 am – 11.00 am
(2 Hours)

| Question | Marks |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| | |
| Total | |

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**INSTRUCTION TO STUDENT**

1. This question paper consists of 13 printed pages (inclusive of the front page) with 4 questions only.
2. Attempt **ALL FOUR** questions. Print your answer on the question paper. The distribution of the marks for each question is given.
3. Show **ALL** of your working steps clearly.

## QUESTION 1 (10 MARKS)

(a) Use truth table to determine whether the statement is tautology, contradiction, or contingency.

$p \rightarrow q \wedge \neg p \vee q$ [2.5 marks]

(b) Show that the propositions $(q \rightarrow \neg p) \wedge (p \rightarrow \neg r)$ and $p \rightarrow \neg(q \vee r)$ are logically equivalent by using equivalence laws. [2.5 marks]

... Continued

(c) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(P(x) \rightarrow S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$, and $\exists x(\neg Q(x))$ are true, then $\exists x(\neg R(x))$ is true.

[5 marks]

··· **Continued**

## QUESTION 2 (15 MARKS)

(a) Let $A$ be set of vowels in the English alphabet and
$B = \{x | x$ is a letter from the word "discrete"$\}$.

(i) List out the unique elements of sets $A$ and $B$. [1 mark]

(ii) Find $A \Delta B$. [1 mark]

(iii) Find $P(A - B)$. [1.5 mark]

(b) Suppose $S$ is a relation on $Z^+$ such that $S = \{(a, b) | a \geq b\}$. Is $S$ transitive? Justify your answer. [2 marks]

(c) Let $f: Z \to Z,\ f(x) = 2x^2$. Is $f$ onto? Justify your answer. [2.5 marks]

... Continued

(d) Use mathematical induction to prove that for all integers $n \geq 1$

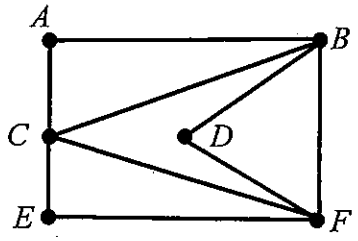
$$\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}=\left(1-\frac{1}{2^n}\right)$$ [5 marks]

(e) Show that $f(x) = 2x^3 + x\,ln\,x$ is $O(x^3)$. [2 marks]

... Continued

## QUESTION 3 (10 MARKS)

(a) Given the following graph:




(i) Find a spanning tree using **breadth-first search** algorithm with $D$ as its root and the selection of neighbouring vertex is according to adjacent vertices with lowest alphabetical order. Clearly show each step of how the algorithm is performed and present your answer in a table with three columns, the first column is the number of steps, the second column is the queue for the search algorithm and the third column is the edges of the resulting spanning tree. [5 mark]

(ii) Determine whether the graph has a Euler circuit. Find the Euler circuit if exists. [2.5 marks]

... Continued

(b) A finite-state automaton $M$ has states $S = \{s_1, s_2, s_3\}$, input values $I = \{a, b\}$, initial state is $s_1$ and the accepting state is $\{s_3\}$. The next state function is given as follows

| State | Next state | |
|---|---|---|
| | $a$ | $b$ |
| $s_1$ | $s_2$ | $s_1$ |
| $s_2$ | $s_3$ | $s_1$ |
| $s_3$ | $s_3$ | $s_1$ |

(i) Construct a state-transition diagram for the automaton $M$. [1.5 marks]

(ii) Use regular expression to describe the language accepted by the automaton $M$. [1 mark]

... Continued

## QUESTION 4 (15 MARKS)

(a) In how many ways can all the letters in SIMULATE be arranged without repetition such that the four vowels must be side by side to one another?

[2 marks]

(b) How many ways are there to assign three jobs to five employees if each employee can be given more than one job? [2 marks]

... Continued

(c) Suppose that 5% of the patients tested in a clinic are infected with avian influenza. Furthermore, suppose that when a blood test for avian influenza is given, 96% of the patients infected with avian influenza test positive and that 2% of the patients not infected with avian influenza test positive.

(i) Express the probabilities in a tree diagram. [1 mark]

(ii) What is the probability that a patient is testing positive for the avian influenza? [2 marks]

(iii) What is the probability that a patient testing negative for avian influenza with this test is infected with it? [2 marks]

**... Continued**

(d) Given is a pair of fair dice.

(i) What is the probability that a total of at least 10 will occurs in a roll?

[1 mark]

(ii) If the pair of dice is rolled 8 times. What is the probability that a total of at least 10 will occurs exactly 3 times? [2 marks]

(iii) If the pair of dice is rolled 400 times. What is the probability that a total of at least 10 will occurs between 70 to 90 times, inclusive? (Show that the Binomial probability can be approximate by Normal distribution and then find the probability.) [3 marks]

**... Continued**

# Appendix A

## List of Rules of Inference

ADD : $p \Rightarrow (p \vee q)$

SIMP : $[p \wedge q] \Rightarrow p$

CONJ : $p \wedge q \Rightarrow (p \wedge q)$

MP : $[(p \rightarrow q) \wedge p] \Rightarrow q$

MT : $[(p \rightarrow q) \wedge (\neg q)] \Rightarrow \neg p$

HS : $[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$

DS : $[(p \vee q) \wedge (\neg p)] \Rightarrow q$

RES : $[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow (q \vee r)$

## List of Logical Equivalence Laws

Conversion of Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Conversion of Equivalence: $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

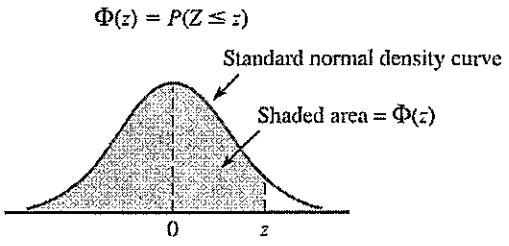
Double Negation: $\neg\neg p \Leftrightarrow p$

| | | | | | |
|---|---|---|---|---|---|
| De Morgan | : | (i) | $\neg (p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$ | | |
| | | (ii) | $\neg (p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$ | | |
| Domination | : | (i) | $p \wedge F \Leftrightarrow F$ | (ii) | $p \vee T \Leftrightarrow T$ |
| Negation | : | (i) | $p \wedge \neg p \Leftrightarrow F$ | (ii) | $p \vee \neg p \Leftrightarrow T$ |
| Identity | : | (i) | $p \wedge T \Leftrightarrow p$ | (ii) | $p \vee F \Leftrightarrow p$ |
| Commutative | : | (i) | $p \wedge q \Leftrightarrow q \wedge p$ | (ii) | $p \vee q \Leftrightarrow q \vee p$ |
| Idempotent | : | (i) | $p \vee p \Leftrightarrow p$ | (ii) | $p \wedge p \Leftrightarrow p$ |
| Distributive | : | (i) | $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ | | |
| | | (ii) | $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ | | |
| Associative | : | (i) | $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \Leftrightarrow p \vee q \vee r$ | | |
| | | (ii) | $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \Leftrightarrow p \wedge q \wedge r$ | | |
| Absorption | : | (i) | $p \vee (p \wedge q) \Leftrightarrow p$ | | |
| | | (ii) | $p \wedge (p \vee q) \Leftrightarrow p$ | | |

**... Continued**

# Appendix B: Standard Normal Curve Areas

Standard Normal Curve Areas




| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| −3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| −3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| −3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0017 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0038 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0352 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0722 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| −1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| −0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| −0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| −0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| −0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| −0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| −0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| −0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3482 |
| −0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| −0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| −0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

**Continued ...**

## Standard Normal Curve Areas

$\Phi(z) = P(Z \leq z)$

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9278 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

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